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Fundamental Research on Gear Noise and Vibration*

(6th Report, Generation Mechanism of Radial and Axial Vibration of Spur Gears)

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The generation mechanism of the radial vibration and the axial one of spur gears is theoretically analyzed in consideration of not only the stiffness of shafts and bearings but also the mass of shafts in this report, where the concerned system of an infinite degree-of-freedom is converted to that of a finite one which has the same kinetic energy as the former, based on the assumption that the static characteristics of shafts and bearings can be applied to the dynamic condition.

This analysis and the comparison of the calculated results based on it with the experimental ones indicate that the vibration model, represented by a system of coupled vibration, can give a comparatively good explanation of the characteristics of three-directional vibrations obtained experimentally and that the conversion ratios from the circumferential vibration to the radial and the axial one depend on the combination of gears, shafts and bearings.

1. Introduction

The 4th report[1] has already shown experimentally that gears vibrate radially and axially as well as circumferentially and that the radial vibration and the axial one give direct effect on the gear noise of gearings without cases. The experiments also show that RMS values of these two-directional vibrations are almost directly proportional to that of the circumferential one and that the fundamental frequency of the former is equal to that of the latter. Therefore, it was supposed that these two-directional vibrations are induced by the circumferential one and that it might be necessary, in order to decrease the gear noise, a) to decrease the conversion ratios from the circumferential vibration to the radial one and to the axial one by clarifying the inducing process of these vibrations as well as b) to decrease the circumferential vibration itself which is the very first generating source of the gear vibrations.

By the way, keeping an eye on only a pair of mating gears, the radial vibration and the axial one should not be generated theoretically. So it is necessary to investigate a system including gear-shafts and bearings, too. The vibration of shafts has been discussed in almost all books concerning the mechanical vibration so far, but most of them put stress on getting the critical speed or the natural frequency of rather simple objects, for instance, of a shaft with uniform cross section or of a massless shaft with concentrated masses or of a shaft with a concentrated mass located in the middle of bearings or on the free end of a canti-lever shaft. T. Yamamoto[2] has investigated the vibration of rotatory machines in detail but the mass of the shaft seems to be neglected.

In this report, the generation mechanism of the radial vibration and the axial one of spur gears is theoretically analyzed in consideration of not only the stiffness of shafts and bearings but also the mass of shafts based on the assumption that the static characteristics of displacements of shafts and bearings can be applied to the dynamic condition (i. e., the vibratory mode is determined previously). On this assumption, the concerned system of an infinite degree of freedom composed of a pair of gears, shafts and bearings is converted to the system of a finite one composed of a

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rigid body and several numbers of springs, which has the same kinetic energy as the former.

In the last part of this paper, the calculated results are compared with the experimental ones.

2. Theoretical analysis

The system including a gear, a shaft and bearings is a complicated mechanical system, where the gear bends as well as deflects. As in Fig. 1, denoting the linear displacement and the rotational one at an arbitrary point B by $y(z_1)$ and $\phi(z_1)$, respectively, when the force P_A and the moment M_A act at the gear position A, then the following relations are obtained :

$$\left.\begin{aligned} y(z_1) &= G_{uu}(z_1,z)P_A+G_{u\theta}(z_1,z)M_A \\ \phi(z_1) &= G_{\theta u}(z_1,z)P_A+G_{\theta\theta}(z_1,z)M_A \end{aligned}\right\} \quad \cdots\cdots\cdots(1)$$

where $G(z_1, z)$'s are the influence functions, whose former suffixes u and θ mean the linear displacement and the rotational one and the latter the force and the moment, respectively. For example, $G_{uu}(z_1, z)$ is the linear displacement at the point B caused by the unit force at A. Setting $z_1=z$ in Eq. (1),

$$y(z)=G_{uu}(z,z)P_A+G_{u\theta}(z,z)M_A\equiv\alpha P_A+\beta M_A$$
$$\phi(z)=G_{\theta u}(z,z)P_A+G_{\theta\theta}(z,z)M_A\equiv\beta P_A+\gamma M_A$$
$$\left.\begin{aligned} P_A &= \{\gamma y(z)-\beta\phi(z)\}/\Delta \\ M_A &= \{-\beta y(z)+\alpha\phi(z)\}/\Delta \\ \Delta &\equiv \alpha\gamma-\beta^2 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(2)$$

where α, β and γ are so-called influence coefficients.

Substituting Eq. (2) in Eq. (1), $y(z_1)$ and $\phi(z_1)$ are expressed by $y(z)$ and $\phi(z)$ as follows, respectively :

$$\left.\begin{aligned} y(z_1) &= [\{\gamma G_{uu}(z_1,z)-\beta G_{u\theta}(z_1,z)\}y(z) \\ &\quad -\{\beta G_{uu}(z_1,z)-\alpha G_{u\theta}(z_1,z)\}\phi(z)]/\Delta \\ \phi(z_1) &= [\{\gamma G_{\theta u}(z_1,z)-\beta G_{\theta\theta}(z_1,z)\}y(z) \\ &\quad -\{\beta G_{\theta u}(z_1,z)-\alpha G_{\theta\theta}(z_1,z)\}\phi(z)]/\Delta \end{aligned}\right\}$$
$$\cdots\cdots\cdots\cdots\cdots(3)$$

Now, according to T. Yamamoto[2], when the horizontal axis (H), the vertical one (V) and the rotational one (Θ) compose the right handed system, the kinetic energy T of a rotor without any unbalances is represented by

$$2T=J\dot\theta^2+m(\dot y_V{}^2+\dot y_H{}^2)+I(\dot\phi_V{}^2+\dot\phi_H{}^2)$$
$$+J\dot\theta(\dot\phi_H\phi_V-\dot\phi_V\phi_H)\cdots\cdots\cdots\cdots\cdots\cdots\cdots(4)$$

where θ is the rotational angle and the displacements y and ϕ are those at the center of gravity of the rotor. The 4th term is a kinetic energy due to the gyroscopic action.

Applying Eq. (4) to the system shown in Fig.1, J, m and I in the equation mean just the polar moment of inertia, the mass and the moment of inertia of the gear, respectively, when the mass of the shaft is negligible. But it is not so easy to get the values when the mass of the shaft must be taken into consideration, because the center of gravity of this system does not always coincide with that of the gear or because the displacements y and ϕ vary with the position, i. e., they are functions of z_1.

It is not of course impossible to get formal solutions for the differential equations of motion of the infinite degree-of-freedom by the method of series expansion, but this method is not practical and there may remain little possibility to obtain the steady state solutions for practical use. Moreover, what is the most interesting is not the steady state vibration at the arbitrary point B but that at the gear position A. Additionally, there remains some interest in the vibration at the bearing position ($z_1=0$ or $z_1=1$), but the accuracy of the value is a little doubtful because the mass of bearings is formally neglected as mentioned later. From such a point of view, fixing eyes on the fact that Eq. (4) is applicable to the infinitesimal element of a rotor, we try to make the degree-of-freedom of the system finite by expressing the kinetic energy T in terms $\dot\theta(z)$, $\dot y(z)$, $\dot\phi(z)$ and so on which define the motion of the position A.

Referring to Eq. (4), the kinetic energy of the infinitesimal element at point B, $\Delta T(z_1)$, is represented as follows :

$$\left.\begin{aligned} \Delta T(z_1) &= \Delta T_\theta(z_1)+\Delta T_{yV}(z_1)+\Delta T_{yH}(z_1) \\ &\quad +\Delta T_{\phi V}(z_1)+\Delta T_{\phi H}(z_1)+\Delta T_g(z_1) \\ \Delta T_\theta(z_1) &= \Delta J(z_1)\{\dot\theta(z_1)\}^2/2 \\ \Delta T_{yi}(z_1) &= \Delta m(z_1)\{\dot y_i(z_1)\}^2/2, \quad i=V,\ H \\ \Delta T_{\phi i}(z_1) &= \Delta I(z_1)\{\dot\phi_i(z_1)\}^2/2, \quad i=V,\ H \\ \Delta T_g(z_1) &= \Delta J(z_1)\dot\theta(z_1)\{\dot\phi_H(z_1)\phi_V(z_1) \\ &\quad -\dot\phi_V(z_1)\phi_H(z_1)\}/2 \end{aligned}\right\}$$
$$\cdots\cdots\cdots\cdots\cdots(5)$$

where $\Delta J(z_1)$, $\Delta m(z_1)$ and $\Delta I(z_1)$ are the polar moment of inertia, the mass and the moment of inertia of the element, respectively.

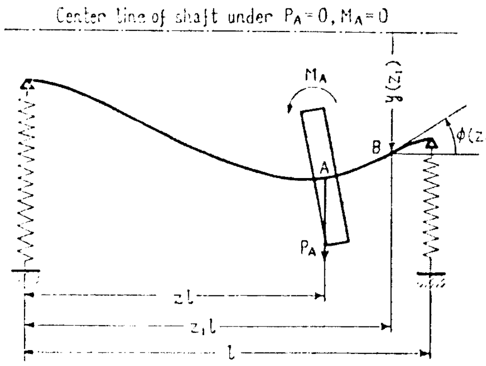


Fig. 1 Displacements of shaft and bearings due to force and moment

Although Eq. (3) comes into existence only under the static conditions in a strict sence, supposing the natural frequency of bearings is high enough to neglect the mass of the bearings, the influence functions G's are considered to be independent of time or, in other words, Eq. (3) could be also applied to the dynamic conditions. Additionally, the following assumption should be reasonable :

$$\theta(z_1)=\theta(z) \qquad \dot{\theta}(z_1)=\dot{\theta}(z) \quad \cdots\cdots(6)$$

Then, what is dependent on z_1 in Eq. (5) comes to be all known or represented by the terms defining the motion of the position A. It is therefore possible to integrate Eq. (5) from $z_1=0$ to $z_1=1$ and to express the kinetic energy T of the whole system only by the terms at the position A. Namely, substituting Eq. (3), its derivatives with respect to time and Eq. (6) into Eq. (5), each term in Eq. (5) is written as follows :

$$\left.\begin{aligned}
&2\Delta T_\theta(z_1)=\{\rho l Z(z_1)\}\dot{\theta}^2\Delta z_1\\
&2\Delta T_{yi}(z_1)=\{\rho l S(z_1)\}\{Y_{yyi}(z_1,z)\dot{y}_i^2-2Y_{y\phi i}(z_1,z)\dot{y}_i\dot{\phi}_i+Y_{\phi\phi i}(z_1,z)\dot{\phi}_i^2\}\Delta z_1, \quad i=V,H\\
&2\Delta T_{\phi i}(z_1)=\{\rho l Z(z_1)/2\}\{\Phi_{yyi}(z_1,z)\dot{y}_i^2-2\Phi_{y\phi i}(z_1,z)\dot{y}_i\dot{\phi}_i+\Phi_{\phi\phi i}(z_1,z)\dot{\phi}_i^2\}\Delta z_1, \quad i=V,H\\
&2\Delta T_g(z_1)=\{\rho l Z(z_1)\}\dot{\theta}\{H_1(\dot{\phi}_H\phi_V-\dot{\phi}_V\phi_H)-H_2(y_H\dot{\phi}_V-\dot{\phi}_V y_H)\\
&\qquad -H_3(\dot{\phi}_H y_V-\dot{y}_V\phi_H)+H_4(\dot{y}_H y_V-\dot{y}_V y_H)\}\Delta z_1
\end{aligned}\right\} \cdots\cdots(7\cdot a)$$

where ρ, $Z(z_1)$ and $S(z_1)$ are the density, the polar moment of inertia of area and the area of the cross section, respectively, and (z) is abbreviated for simplification, such as $\dot{\theta}(z)\to\dot{\theta}$, and

$$\begin{pmatrix} Y_{yyi}, & \Phi_{yyi}\\ Y_{y\phi i}, & \Phi_{y\phi i}\\ Y_{\phi\phi i}, & \Phi_{\phi\phi i}\end{pmatrix}=\frac{1}{\Lambda_i^2}\begin{pmatrix} \gamma_i^2, & 2\beta_i\gamma_i, & \beta_i^2\\ \beta_i\gamma_i, & \alpha_i\gamma_i+\beta_i^2, & \alpha_i\beta_i\\ \beta_i^2, & 2\alpha_i\beta_i, & \alpha_i^2\end{pmatrix}\begin{pmatrix} G_{uui}^2(z_1,z) & G_{\theta ui}^2(z_1,z)\\ -G_{uui}(z_1,z)G_{u\theta i}(z_1,z) & -G_{\theta ui}(z_1,z)G_{\theta\theta i}(z_1,z)\\ G_{u\theta i}^2(z_1,z) & G_{\theta\theta i}^2(z_1,z)\end{pmatrix}$$

$$i=V,\ H \cdots\cdots(7\cdot b)$$

$$\left.\begin{aligned}
&H_1=H_{\beta\alpha V}H_{\beta\alpha H},\ H_2=H_{\beta\alpha V}H_{\gamma\beta H},\ H_3=H_{\gamma\beta V}H_{\beta\alpha H},\ H_4=H_{\gamma\beta V}H_{\gamma\beta H}\\
&H_{\beta\alpha i}=\{\beta_i G_{\theta ui}(z_1,z)-\alpha_i G_{\theta\theta i}(z_1,z)\}/\Lambda_i,\ H_{\gamma\beta i}=\{\gamma_i G_{\theta ui}(z_1,z)-\beta_i G_{\theta\theta i}(z_1,z)\}/\Lambda_i
\end{aligned}\right\} \cdots\cdots(7\cdot c)$$

Equation (7) makes it clear that the vertical displacements and the horizontal ones are not coupled dynamically with the exception of displacements due to the gyroscopic action. So integrating Eq. (5) from 0 to 1 with respect to z_1 except for $\Delta T_g(z_1)$ results in the kinetic energy T of the whole system as follows :

$$2T=J\dot{\theta}^2+\sum_{i=V,H}(m_i\dot{y}_i^2+I_i\dot{\phi}_i^2-2C_i\dot{y}_i\dot{\phi}_i) \cdots(8)$$

where J is the polar moment of inertia of the gear and the shaft, and m, I and C are so-called equivalent mass, equivalent moment of inertia and dynamic coupling coefficient between y and ϕ, respectively. These are functions of the gear position z and written as follows :

$$\left.\begin{aligned}
m_i&=\rho l\int_0^1\{Y_{yyi}(z_1,z)S(z_1)\\
&\qquad+\Phi_{yyi}(z_1,z)Z(z_1)/2\}dz_1\\
I_i&=\rho l\int_0^1\{Y_{\phi\phi i}(z_1,z)S(z_1)\\
&\qquad+\Phi_{\phi\phi i}(z_1,z)Z(z_1)/2\}dz_1\\
C_i&=\rho l\int_0^1\{Y_{y\phi i}(z_1,z)S(z_1)\\
&\qquad+\Phi_{y\phi i}(z_1,z)Z(z_1)/2\}dz_1
\end{aligned}\right\} i=V,H \cdots\cdots(9)$$

Equation (9) proves that each value of m, I and C in direction of V does not coincide in general with one in direction of H.

By the way, a pair of mating gears is acted upon by such forces and moments as in Fig. 2, where suffixes 1 and 2 represent the driving gear and the driven one, respectively. O' is the center of the gear and the rotating axis of the gear coincides with the axis OΘ when neither the linear displacement OO' nor the rotational ones ϕ_V and ϕ_H exist. The symbols used in Fig. 2 are as follows :

r_g : radius of base circle, α_b : pressure angle

T_{Q1}, T_{Q2} : driving torque and driven one

W_d, M_d : normal force or dynamic load and tangential moment acting between gear faces in contact

P_V, P_H : reacting force of shaft on gear in directions of V and H

M_V, M_H : reacting moment of shaft on gear about axes V' and H' which are parallel to axes V and H, respectively

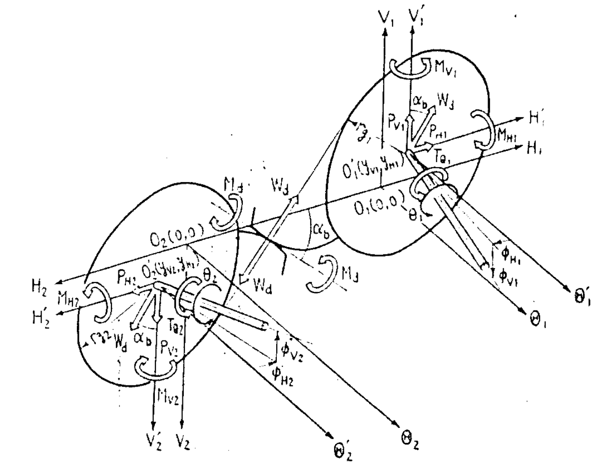


Fig. 2 Forces and moments acting on a pair of gears

Equation (8) is applicable to both of the driving system and the driven one. Balancing the forces and the moments shown in Fig. 2 with the inertia forces obtained by substituting Eq. (8) into Lagrangian equations, the following equations of motion are derived :

$$J_i\ddot{\theta}_i=T_{Qi}-W_d r_{gi} \quad i=1,2 \quad \cdots\cdots(10)$$

$$\left.\begin{aligned}
&m_{Vi}\ddot{y}_{Vi}-C_{Vi}\ddot{\phi}_{Vi}=W_d\cos\alpha_b+P_{Vi}\\
&m_{Hi}\ddot{y}_{Hi}-C_{Hi}\ddot{\phi}_{Hi}=W_d\sin\alpha_b+P_{Hi}\\
&I_{Vi}\ddot{\phi}_{Vi}-C_{Vi}\ddot{y}_{Vi}=-M_d\cos\alpha_b+M_{Hi}\\
&I_{Hi}\ddot{\phi}_{Hi}-C_{Hi}\ddot{y}_{Hi}=-M_d\sin\alpha_b-M_{Vi}
\end{aligned}\right\} \quad i=1,2 \cdots\cdots(11)$$

where the friction forces are neglected and the damping forces are omitted.

The static load acting normally between gear faces W is

$$W=T_{Q1}/r_{g1}=T_{Q2}/r_{g2}$$

and the equations of motion with respect to θ_1 and θ_2 expressed by Eq. (10) result in the following equation in same way as mentioned in the 1st report[3] :

$$M\ddot{x}(=M_1\ddot{x}_1=M_2\ddot{x}_2)=W-W_d \quad \cdots\cdots(10)'$$

where

$$M\equiv M_1M_2/(M_1+M_2),\ M_1\equiv J_1/r_{g1}^2,$$
$$M_2\equiv J_2/r_{g2}^2,\ x\equiv x_1+x_2,\ x_1\equiv r_{g1}\theta_1,\ x_2\equiv r_{g2}\theta_2$$

The dynamic force W_d and the dynamic moment M_d acting between gear faces are produced by the displacement of gear teeth. Assuming for simplification that the displacement of gear teeth is distributed linearly over the gear tooth width b, as in Fig. 3, the following expressions are reasonable :

$$\left.\begin{aligned}
&W_d=K(t,\delta,\varphi b)\delta-F(t,\delta,\varphi b)\\
&\delta=x-(y_{V1}+y_{V2})\cos\alpha_b-(y_{H1}+y_{H2})\sin\alpha_b\\
&M_d=K(t,\delta,\varphi b)b^2\varphi/12\\
&\varphi=(\phi_{V1}+\phi_{V2})\cos\alpha_b+(\phi_{H1}+\phi_{H2})\sin\alpha_b-\varphi_g
\end{aligned}\right\} \cdots\cdots(12)$$

where

K : stiffness of mating gear teeth

F : disturbing force due to gear tooth profile errors

δ : linear displacement of mating gear teeth in direction of line-of-action, measured in the middle of gear tooth width b

φ : rotational displacement of mating gear teeth about normal to line-of-action

φ_g : sum of errors in angle, such as lead error or error in parallelism of shafts, about normal to line-of-action

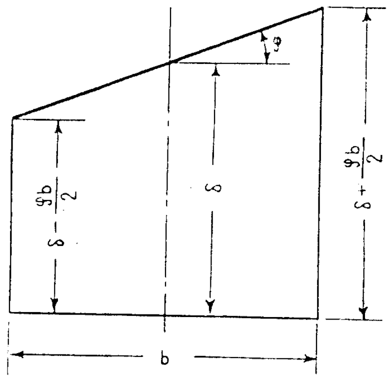


Fig. 3 Displacement distribution of gear teeth

Moreover, both the stiffness K and the disturbing force F are functions not only of time t but also of δ and φb, because gear teeth have a chance to be out of contact partially or wholly according to the sign of $\delta\pm\varphi b/2$, even if they are in contact geometrically.

The reacting forces P_V, P_H and the reacting moments M_V, M_H are derived by changing the sign of Eq. (2). So they are rewritten as follows, referring to the direction of the force and the moment indicated in Figs. 1 and 2 :

$$\left.\begin{aligned}
&P_{Vi}=-(\gamma_{Vi}y_{Vi}-\beta_{Vi}\phi_{Vi})/\Lambda_{Vi}=-y_{Vi}/\alpha_{Vi}\\
&\qquad-(\beta_{Vi}/\alpha_{Vi})(\beta_{Vi}y_{Vi}-\alpha_{Vi}\phi_{Vi})/\Lambda_{Vi}\\
&P_{Hi}=-y_{Hi}/\alpha_{Hi}-(\beta_{Hi}/\alpha_{Hi})\\
&\qquad\times(\beta_{Hi}y_{Hi}-\alpha_{Hi}\varphi_{Hi})/\Lambda_{Hi}\\
&M_{Hi}=(\beta_{Vi}y_{Vi}-\alpha_{Vi}\phi_{Vi})/\Lambda_{Vi}\\
&M_{Vi}=-(\beta_{Hi}y_{Hi}-\alpha_{Hi}\phi_{Hi})/\Lambda_{Hi}
\end{aligned}\right\} \quad i=1,2\cdots\cdots(13)$$

Substituting Eq. (13) in Eq. (11), we have the equations of motion for the radial vibration and the axial one in case of neglecting the gyroscopic action. Equations (11) and (13) indicate that equations for the driving system are identical in form with those for the driven one, and so omitting suffixes 1 and 2, the differential equations of motion are expressed as follows :

$$\left.\begin{aligned}
&m_V\ddot{y}_V-C_V\ddot{\phi}_V+y_V/\alpha_V+(\beta_V/\alpha_V)\\
&\quad\times(\beta_V y_V-\alpha_V\phi_V)/\Lambda_V=W_d\cos\alpha_b\\
&m_H\ddot{y}_H-C_H\ddot{\phi}_H+y_H/\alpha_H+(\beta_H/\alpha_H)\\
&\quad\times(\beta_H y_H-\alpha_H\phi_H)/\Lambda_H=W_d\sin\alpha_b\\
&I_V\ddot{\phi}_V-C_V\ddot{y}_V+(\alpha_V\phi_V-\beta_V y_V)/\Lambda_V\\
&\quad=-M_d\cos\alpha_b\\
&I_H\ddot{\phi}_H-C_H\ddot{y}_H+(\alpha_H\phi_H-\beta_H y_H)/\Lambda_H\\
&\quad=-M_d\sin\alpha_b
\end{aligned}\right\} \cdots\cdots(11)'$$

When the driven system is identical to the driving one, the following expressions for W_d and M_d will be reasonable instead of Eq. (12) :

$$\left.\begin{aligned}
&W_d=K(x-2y_V\cos\alpha_b-2y_H\sin\alpha_b)-F\\
&M_d=Kb^2(\phi_V\cos\alpha_b+\phi_H\sin\alpha_b-\varphi_g/2)/6
\end{aligned}\right\} \cdots\cdots(12)'$$

Supposing, in addition, the directions V and H are of the same mechanical system, it is clear from Eq. (11)$'$ that $y_V/\cos\alpha_b=y_H/\sin\alpha_b\equiv y$, ϕ_V

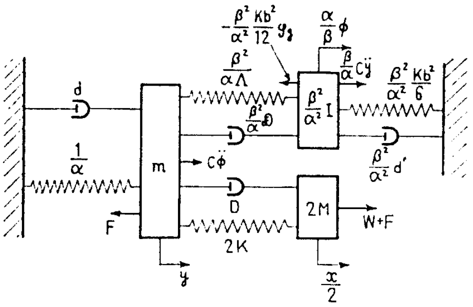


Fig. 4 Equivalent vibration system of three directional vibration of gears

$/\cos\alpha_b=\phi_H/\sin\alpha_b\equiv\phi$, and then the differential equations of motion with respect to y and ϕ are expressed in the following form:

$$\left.\begin{aligned}&m\ddot{y}-C\ddot{\phi}+y/\alpha+(\beta/\alpha)(\beta y-\alpha\phi)/\Lambda\\&\quad=K(x-2y)-F\\&I\ddot{\phi}-C\ddot{y}+(\alpha\phi-\beta y)/\Lambda+Kb^2\phi/6\\&\quad=Kb^2\varphi_g/12\end{aligned}\right\}\cdots(11)''$$

Equation (11)'' proves that the linear displacement is induced only in direction of line-of-action and that the rotational one only in direction about normal to line-of-action, when there exists perfect identity between the driving system and the driven one and between the system in V direction and the one in H.

The vibration system expressed by Eq. (11)'' is schematically indicated in Fig. 4 including the component x, where appropriate viscous dampers analogous to the corresponding stiffness are added. Figure 4 makes it clear that the system of three-directional vibration of gears is represented by a coupled vibration system where the circumferential vibration (x) and the radial one (y) are mutually coupled statically and the radial vibration and the axial one (ϕ) are coupled both statically and dynamically. It is also clear that the coupling coefficients are determined by the gear teeth stiffness (K) or several elastic constants of the shaft-bearings system (α, β, γ and Λ).

Using expression (12)' and adding proper damping terms to Eqs. (10)' and (11)', we have

$$\left.\begin{aligned}&M\ddot{x}+D(\dot{x}-2\dot{y}_V\cos\alpha_b-2\dot{y}_H\sin\alpha_b)+K(x-2y_V\cos\alpha_b-2y_H\sin\alpha_b)=W+F\\&m_V\ddot{y}_V+d_V\dot{y}_V+y_V/\alpha_V+(\beta_V/\alpha_V)\{\mathfrak{D}_V(\beta_V\dot{y}_V-\alpha_V\dot{\phi}_V)+(\beta_Vy_V-\alpha_V\phi_V)/\Lambda_V\}\\&\qquad=(W-M\ddot{x})\cos\alpha_b+C_V\ddot{\phi}_V\\&m_H\ddot{y}_H+d_H\dot{y}_H+y_H/\alpha_H+(\beta_H/\alpha_H)\{\mathfrak{D}_H(\beta_H\dot{y}_H-\alpha_H\dot{\phi}_H)+(\beta_Hy_H-\alpha_H\phi_H)/\Lambda_H\}\\&\qquad=(W-M\ddot{x})\sin\alpha_b+C_H\ddot{\phi}_H\\&I_V\ddot{\phi}_V+\{d'(\dot{\phi}_V\cos\alpha_b+\dot{\phi}_H\sin\alpha_b)+(Kb^2/6)(\phi_V\cos\alpha_b+\phi_H\sin\alpha_b)\}\\&\qquad\times\cos\alpha_b+\mathfrak{D}_V(\alpha_V\dot{\phi}_V-\beta_V\dot{y}_V)+(\alpha_V\phi_V-\beta_Vy_V)/\Lambda_V=Kb^2\varphi_g\cos\alpha_b/12+C_V\ddot{y}_V\\&I_H\ddot{\phi}_H+\{d'(\dot{\phi}_V\cos\alpha_b+\dot{\phi}_H\sin\alpha_b)+(Kb^2/6)(\phi_V\cos\alpha_b+\phi_H\sin\alpha_b)\}\\&\qquad\times\sin\alpha_b+\mathfrak{D}_H(\alpha_H\dot{\phi}_H-\beta_H\dot{y}_H)+(\alpha_H\phi_H-\beta_Hy_H)/\Lambda_H=Kb^2\varphi_g\sin\alpha_b/12+C_H\ddot{y}_H\end{aligned}\right\}\cdots\cdots(14)$$

where D, d, $\mathfrak{D}$ and d' are damping coefficients.

Equation (14) is a set of differential equations for three-directional vibration of gear system in which the driving system is identical with the driven one and the gyroscopic action is neglected.

In like manner, the terms to be changed in Eq. (14) in case of considering the gyroscopic action are as follows:

$$\left.\begin{aligned}&M\ddot{x}\rightarrow M\ddot{x}-Mr_g\{H_1'(\ddot{\phi}_V\phi_H-\ddot{\phi}_H\phi_V)-H_2'(\ddot{\phi}_Vy_H-\ddot{y}_H\phi_V)\\&\qquad-H_3'(\ddot{y}_V\phi_H-\ddot{\phi}_Hy_V)+H_4'(\ddot{y}_Vy_H-\ddot{y}_Hy_V)\}\\&m_V\ddot{y}_V\rightarrow m_V\ddot{y}_V+Mr_g\{H_3'(\ddot{x}\phi_H+2\dot{x}\dot{\phi}_H)-H_4'(\ddot{x}y_H+2\dot{x}\dot{y}_H)\}/2\\&m_H\ddot{y}_H\rightarrow m_H\ddot{y}_H-Mr_g\{H_2'(\ddot{x}\phi_V+2\dot{x}\dot{\phi}_V)-H_4'(\ddot{x}y_V+2\dot{x}\dot{y}_V)\}/2\\&I_V\ddot{\phi}_V\rightarrow I_V\ddot{\phi}_V-Mr_g\{H_1'(\ddot{x}\phi_H+2\dot{x}\dot{\phi}_H)-H_2'(\ddot{x}y_H+2\dot{x}\dot{y}_H)\}/2\\&I_H\ddot{\phi}_H\rightarrow I_H\ddot{\phi}_H+Mr_g\{H_1'(\ddot{x}\phi_V+2\dot{x}\dot{\phi}_V)-H_3'(\ddot{x}y_V+2\dot{x}\dot{y}_V)\}/2\\&H_i'=(\rho l/2Mr_g^2)\int_0^1H_iZ(z_1)dz_1\qquad i=1,\cdots\cdots,4\end{aligned}\right\}\cdots\cdots(15)$$

In Eq. (15), H_1' is of the order 1 and the other $H's$ are much less than H_1'.

In the following, some considerations are made regarding the stability of three-directional vibration of gears. Applying the discussion in our 5th report[4], the motions expressed by Eq. (14) or its modified equation by Eq. (15) are stable if all the free vibrations of x, y and ϕ tend to zero with the time t. By the way, the homogeneous differential equations of Eq. (14), which express the free vibrations, are all represented in the following form of the parameter-excited-vibration:

$$\ddot{u}+q(t)\dot{u}+\omega^2(t)u=0\qquad q(t+T_z)=q(t),\quad \omega^2(t+T_z)=\omega^2(t)$$

where T_z is a period of contact. Therefore, the stability judgement chart in the 5th report is applicable as it is. In the equation of free vibration for x, the stiffness is $K(t)$, and the stiffness $k(t)$ concerning

y and ϕ is written in the form

$$k(t)=cK(t)+k_0$$

where c and k_0 are neither less than 0 nor dependent on time t. Accordingly, we have the relation

$$(k_{ii}-k_i)/\bar{k} \leqq (K_{ii}-K_i)/\bar{K}$$

where k_i, k_{ii}, $\bar{k}$ and K_i, K_{ii}, $\bar{K}$ are the averaged values of $k(t)$ and $K(t)$ over the single contact region, the double contact region and the period of contact, respectively. On the other hand, the chart for stability judgement in the 5th report indicates that the smaller the specific stiffness of $(K_{ii}-K_i)/\bar{K}$, the further away from the unstable region the circumferential vibration (x), and that the circumferential vibration is stable when a moderate damping exists. Therefore, the stability of the vibration x is, in general, a necessary and sufficient condition for the stability of the vibrations y and ϕ, if there is not a great difference among each damping coefficient of three-directional vibrations. The vibration x is generally stable as mentioned above, and so the three-directional vibration can be concluded to be stable.

To conclude this section, we refer to the formulas for the influence functions $G(z_1, z)$ and the influence coefficients α, β and γ. They can be represented as follows, when 2 bearings at each side of a shaft have the same elastic properties, i. e., the stiffness is equal to K_p and the degree of hinged (or simple) support is $(1-\lambda)$ and so the degree of fixed (or built-in) support is λ:

$$\begin{aligned}
G_{uu}(z_1,z) =& [(1-\lambda)(1-z-z_1+2zz_1)+\lambda/2]/K_p \\
&+\lambda(1-2z)(1-2z_1)\{1+2z(1-z)\} \\
&\times\{1+2z_1(1-z_1)\}/2(K_p+K_s) \\
&+\frac{4}{K_s}\left[(1-\lambda)\begin{Bmatrix}(1-z)z_1\{z(2-z)-z_1^2\} \\ (1-z_1)z\{z_1(2-z_1)-z^2\}\end{Bmatrix}\right. \\
&\left.+\lambda\begin{Bmatrix}(1-z)^2z_1^2\{3z-(1+2z)z_1\} \\ (1-z_1)^2z^2\{3z_1-(1+2z_1)z\}\end{Bmatrix} \begin{matrix}: z_1 \leqq z \\ : z_1 \geqq z\end{matrix}\right]
\end{aligned}$$

$$G_{u\theta}(z_1,z)=-\{\partial G_{uu}(z_1,z)/\partial z\}/l$$

$$G_{\theta u}(z_1,z)=-\{\partial G_{uu}(z_1,z)/\partial z_1\}/l=G_{u\theta}(z,z_1)$$

$$\begin{aligned}
G_{\theta\theta}(z_1,z)&=-\{\partial G_{\theta u}(z_1,z)/\partial z\}/l \\
&=\{\partial^2 G_{uu}(z_1,z)/\partial z\partial z_1\}/l^2
\end{aligned}$$

$$\begin{aligned}
\alpha=G_{uu}(z,z)=&(1-\lambda)[\{1-2z(1-z)\}/K_p \\
&+8z^2(1-z)^2/K_s]+\lambda[1/K_p+(1-2z)^2 \\
&\times\{1+2z(1-z)\}^2/2(K_p+K_s)+8z^3(1-z)^3/K_s]
\end{aligned}$$

$$\begin{aligned}
\beta=G_{u\theta}(z,z)=G_{\theta u}(z,z)=&(1-2z)[(1-\lambda) \\
&\times\{1/K_p-8z(1-z)/K_s\}+\lambda[6z(1-z)\{1+2z \\
&\times(1-z)\}^2/(K_p+K_s)-12z^2(1-z)^2/K_s]]/l
\end{aligned}$$

$$\begin{aligned}
\gamma=G_{\theta\theta}(z,z)=&[(1-\lambda)[2/K_p+8\{1-3z(1-z)\} \\
&/K_s]+\lambda[72z^2(1-z)^2/(K_p+K_s)+24z(1-z) \\
&\times\{1-3z(1-z)\}/K_s]]/l^2
\end{aligned}$$

$$K_s=6E\pi(r_0^4-r_i^4)/l^3$$

where K_s : stiffness of shaft, E : Young's modulus of elasticity, r_0, r_i : outside radius and inside one of the shaft, respectively.

According to these equations, $\beta=0$ and $C=0$ when $z=1/2$ or, in other words, the coupling between y and ϕ is released both statically and dynamically when the gear is set just in the middle of bearings, and ϕ has no disturbing force either if no error in angle such as lead error exists in addition. Therefore, $\phi=0$ at the steady state and the axial vibration cannot be induced then.

In the above-mentioned analysis, neither the static unbalance nor the dynamic one is considered, the reason for which is that it is necessary to calculate the periodic solutions over at least one revolution of the gear which is supposed to be almost impossible to make because of the limit in time available to calculate.

3. Method of experiments and test-gears

The apparatus for the experiments is of power absorbing type setting test-gears in an anechoic chamber, whose outline has been already reported in the 1st report. The main object of the experiments is to make clear how the conversion ratios of the radial vibration and the axial one from the circumferential vibration undergo a transition depending on the gear position relative to bearings, for which purpose the experiments were carried out in 2 series as in Fig. 5 : in the series A, test-gears were set at 7 positions by varying the combination of collars ; and in the series B, set at 3 positions by moving the gear-fastening-flange with a slitted taper.

A pair of test-gears used in both series are full-depth standard involute spur gears whose module $m=3$, number of gear teeth $z_1=z_2=36$, tooth width $b=10$mm, pressure angle $\alpha=20°$, top clearance ratio $c_k=0.157$ and whose center distance is set at 108.3mm. Their tooth profile error

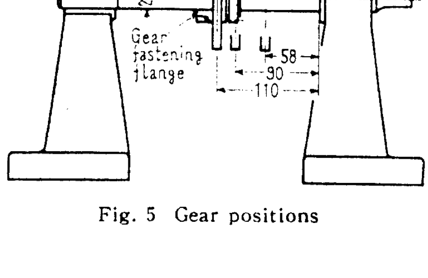


Fig. 5 Gear positions

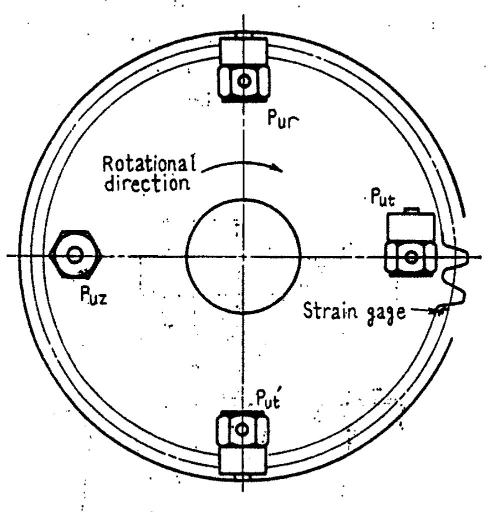
is classified in the quality 1 (JIS) and the other errors of gear in the quality 0 (JIS).

In order to measure the strain at the gear tooth fillet and the accelerations of gear vibrations, a set of 4 wire strain gages and 4 accelerometers of piezo-electric type (B & K type 4335) are fixed to the driven test-gear as in Fig. 6, where the accelerometer P_{ut}' is electrically connected with the accelerometer P_{ut} so as to compensate the radial vibration picked up also by P_{ut}, as pointed out in the 4th report, and it is useful to keep the unbalance as small as possible.

In order to know values required for the theoretical calculations, such as the bearing stiffness K_p, the shaft stiffness K_s and the coefficient of bearing support λ, the relation between the deflections detected by an electric micrometer and the torque by a set of wire strain gages fixed to the shaft is recorded on the $X-Y$ recorder, when the torque is supplied quasi-statically and the test-gears are set at the positions A_1 and B_1, respectively. From its tangent, which was rather difficult to draw because of the hysteresis in the deflection of bearings, the distribution of the specific deflections was averagingly obtained and the values of K_p, K_s and λ as given in Table 1 were obtained.

Table 1 Experimental values for stiffnesses of shaft and bearing

		Series A	Series B
Stiffness of shaft	K_s	9 690kg/mm	1 420kg/mm
Equivalent radius of shaft	r_{eq}	22.6mm	14.0mm
Coefficient of bearing support	λ	0.152	0.50
Stiffness of bearing	Vertical K_{pV}	2 440kg/mm	
	Horizontal K_{pH}	1 430kg/mm	



P_{ut} : accelerometer to measure circumferential vibration
P_{ut}' : accelerometer to correct circumferential vibration
P_{ur} : accelerometer to measure radial vibration
P_{uz} : accelerometer to measure axial vibration
Fig. 6 Arrangement of strain gage and accelerometers

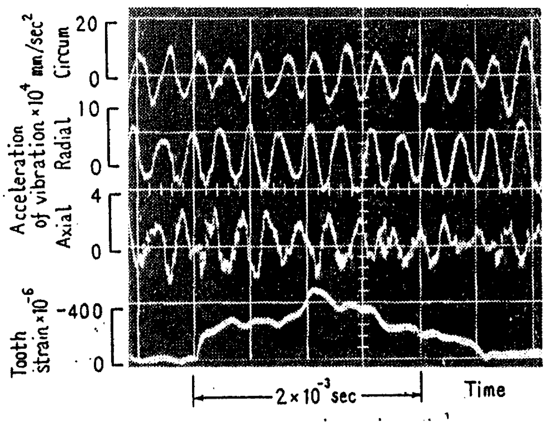
In the series A, the equivalent radius of shaft calculated from the value K_s does not coincide with the actual one (25mm) and λ is smaller, or the degree of support in bearings is nearer to simple support, than that in the series B. These are probably ascribed to the method of fastening gear by means of collars where the shaft does not consist of one body. In addition, the shaft length l is estimated at 220mm (see Fig. 5) but this is controvertible. And it is due to a larger deflection of the bearing-metal than that of the bearing-pedestal that the value K_p has turned out unexpectedly small.

4. Comparison of experimental results with theoretically calculated results and discussion

4·1 Wave form of vibration

Figure 7 indicates an experimental result regarding the wave forms of the accelerations of three-directional vibrations and the gear tooth strain, where the test-gears are positioned at A_3. The corresponding theoretical result and the gear tooth profile errors used in the calculations are shown in Fig. 8, where the gyroscopic action is neglected. Referring to the arrangement of the accelerometers shown in Fig. 6, $-\ddot{x}r/2r_g$, $\ddot{y}_V$ and $r\ddot{\phi}_H$ in Fig. 8 nearly correspond to the circumferential vibration, the radial one and the axial vibration, respectively, when the distance between each accelerometer and the center of the shaft is represented by r. Wave forms of $\ddot{y}_H$ and $r\ddot{\phi}_H$ are additionally indicated in Fig. 8.

In the theoretical calculations, the viscous



Gear position A_3, N=1 000rpm, T_Q=4 000kgmm, Experimental result
Fig. 7 Wave forms of three-directional vibrations and tooth strain

dampings are assumed to be of the type of constant logarithmic decrement as follows :

$$D=2\zeta\sqrt{MK},\ d_V=2\zeta\sqrt{m_V/\alpha_V},\ d_H=2\zeta\sqrt{m_H/\alpha_H}$$
$$\mathfrak{D}_V=2\zeta\sqrt{I_V\alpha_V/\Lambda_V},\qquad \mathfrak{D}_H=2\zeta\sqrt{I_H\alpha_H/\Lambda_H}$$
$$d'=2\zeta\sqrt{\frac{I_VI_H}{I_V\sin^2\alpha_b+I_H\cos^2\alpha_b}\frac{Kb^2}{6}}$$

and each fraction of critical damping ζ is assumed to be 0.07.

Runge-Kutta-Merson's process was employed to solve a set of differential equations and the calculations were executed by KDC II (HITAC 5020) at Kyoto University Computation Center.

Comparing Fig. 8 with Fig. 7, the calculated results agree well with the experimental ones, i. e., the gear oscillates at one main frequency regardless of its vibrational directions, and the axial vibration is in anti-phase with the radial one. According to the theoretically calculated results, this main frequency is not perfectly equal to the natural frequency of the circumferential free vibration but the nearest to it among those of three-directional free vibrations.

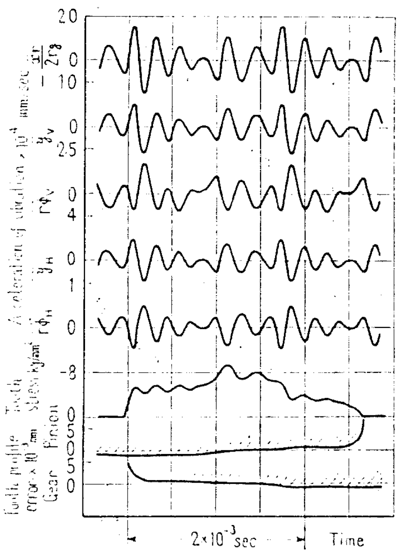
On the other hand, the frequency spectrums of three-directional vibrations are almost identical with each other and $\ddot{y}$ is in phase with $-\ddot{x}$ in the calculated result, while, in the experimental result, the main wave in the axial vibration is overlapped by waves of smaller amplitude and higher frequency, and the radial vibration is slightly out of phase with the circumferential one. This may be mainly the result of the non-linearity in the stiffness of bearings, the elasticity of the gear body itself and the fact that the dynamic properties of the deflections would not be represented by such a simple linear equation as Eq. (3) when the mass of bearings or the frequency response of bearings is taken into consideration even if the stiffness of bearings should be linear.

As is evident from Figs. 7 and 8, the characteristics of the circumferential vibration, such as the periodicity in respect of the contact period T_z, the generation of larger oscillation by so-called contact-impulse and so on, can be also applied to the radial vibration and the axial one.

4·2 Effect of driving speed on RMS values of accelerations of three-directional vibrations

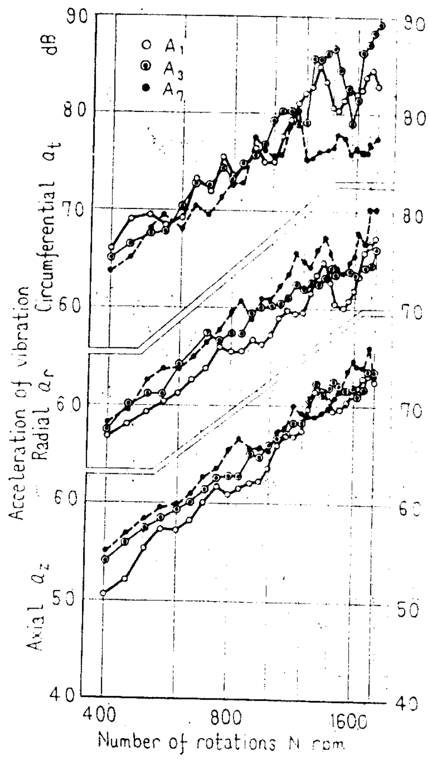
Figure 9, the experimental results for the series A, shows how the RMS values of accelerations of three-directional vibrations a_t, a_r and a_z vary with the driving speed N, where the transmitted torque T_Q=4 000kgmm constant and the test-gears positions are A_1, A_3 and A_7, respectively. And the corresponding calculated results for gear positions A_3 and A_7 are shown in Fig. 10.

Referring to Fig. 6, the vibrations detected by each accelerometer are equal to $-\ddot{x}r/2r_g$, $\ddot{y}_V\cos\Omega t+\ddot{y}_H\sin\Omega t$ and $r(\ddot{\phi}_V\sin\Omega t+\ddot{\phi}_H\cos\Omega t)$, respectively, when the rotational angular velocity is Ω. So the corresponding theoretical RMS values



Gear position A_3, N=1 000rpm, T_Q=4 000kgmm, Calculated result

Fig. 8 Wave forms of three-directional vibrations and tooth stress



T_Q=4 000kgmm, Experimental result

Fig. 9 Transition of RMS value of three-directional vibration with number of rotations are calculated as follows by using their periodicity regarding T_z :

$$a_t=\sqrt{\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}\left(\frac{\ddot{x}r}{2r_g}\right)^2 dt}=\sqrt{\frac{1}{T_z}\int_0^{T_z}\left(\frac{\ddot{x}r}{2r_g}\right)^2 dt}$$

$$a_r=\sqrt{\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}(\ddot{y}_V\cos\Omega t+\ddot{y}_H\sin\Omega t)^2 dt}$$

$$\fallingdotseq\sqrt{\frac{1}{2T_z}\int_0^{T_z}(\ddot{y}_V{}^2+\ddot{y}_H{}^2)dt}$$

$$a_z=\sqrt{\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}r^2(\ddot{\phi}_V\sin\Omega t+\ddot{\phi}_H\cos\Omega t)^2 dt}$$

$$\fallingdotseq\sqrt{\frac{r^2}{2T_z}\int_0^{T_z}(\ddot{\phi}_V{}^2+\ddot{\phi}_H{}^2)dt}$$

and they are indicated by the unit dB, where 10mm/sec^2 is 0 dB.

As is evident from Figs. 9 and 10, the circumferential vibration of the gear varies in its value with the gear position even if the number of rotations and the transmitted torque are identical, and the values of the radial vibration and the axial one show in general the increasing tendency as the gear position approaches a bearing. Although each value of three-directional vibrations increases generally with an increase in the number of rotations, there accompany some rises and falls. This is not different from the discussions already

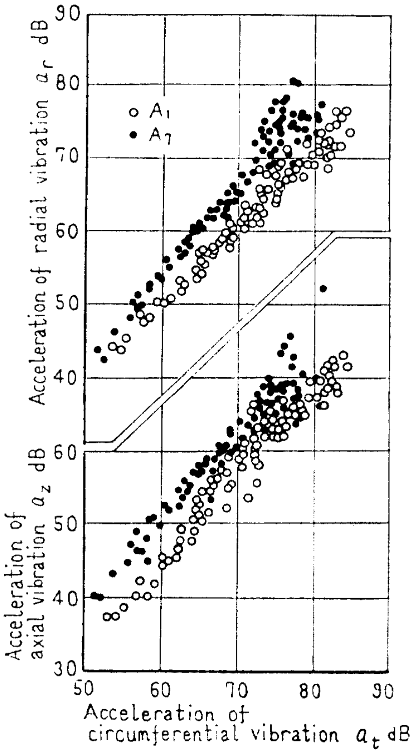
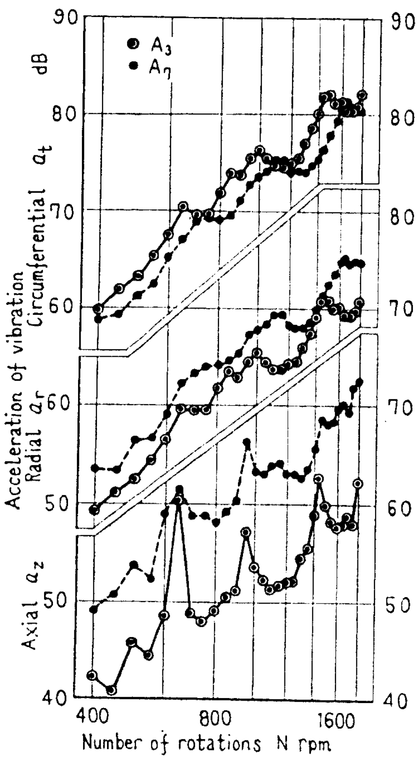


Fig. 11 Relation among RMS values of three-directional vibrations (Experimental result)



T_Q=4 000kgmm, Calculated result

Fig. 10 Transition of RMS value of three-directional vibration with number of rotations

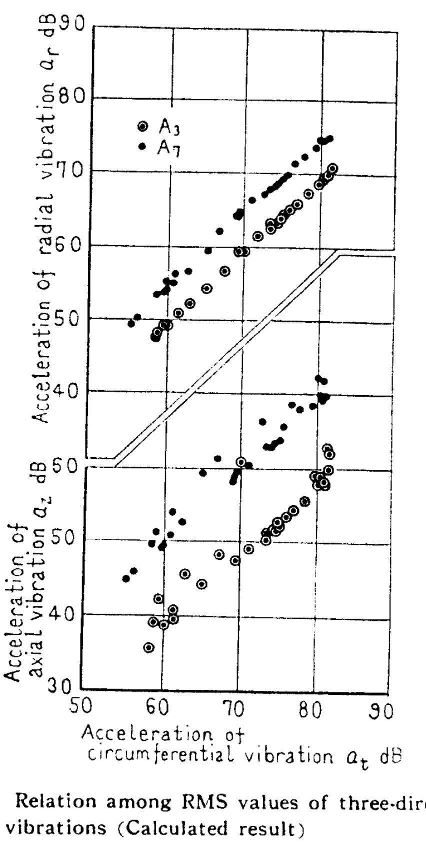


Fig. 12 Relation among RMS values of three-directional vibrations (Calculated result)

reported in the 2nd report[5] on the effect of rotational speed on the circumferential vibration.

According to the theoretically calculated results in Fig. 10, the main frequency is clearly higher in A_7 than in A_3 and both curves for the circumferential vibration coincide with each other if one curve is parallelly shifted. On the other hand, the similar tendency may be recognized in the region of $N=1000\sim1500$rpm in the experiments as in Fig. 9, but not so clearly, and the behaviours in case of A_7 are different especially in the region of $N\geqq1200$rpm, which may be ascribed to the effect of vibrations of bearings considering the closeness of test-gears to a bearing.

In addition, the larger peak of the axial vibration at $N=650$rpm in A_3 results from the 3rd harmonic resonance between $\omega_{\phi r}$, the natural frequency of ϕ_r, and the contact frequency f_z, i.e., the relation $\omega_{\phi r}\fallingdotseq 3f_z$ comes into existence then.

Comparing Fig. 10 with Fig. 9 as a whole, a perfect coincidence in detail between theoretical results and the experimental ones cannot be expected mainly because of the fairly rough assumptions in the theoretical analysis, but it may be quite all right to consider that the general tendency is clarified by the calculations.

As for the effect of the gyroscopic action, the difference in RMS values between the calculated results neglecting this effect and those considering it is as little as ±0.5dB and any special change in wave forms cannot be recognized either.

So it may be concluded that the gyroscopic action can be almost ignored in gears being operated at much lower speed than the critical speed of the gear-shaft.

4·3 Proportional relation among three-directional vibrations (Conversion ratio)

In the 4th report, the experiments have clarified that there exists a direct proportional relation among RMS values of three-directional vibrations and the constants of proportionality are defined as the conversion ratios.

Figure 11 shows this fact, where the RMS values of the radial vibration a_r and those of the axial one a_z are plotted for those of the circumferential one a_t using the experimental results for A_1 and A_7. Figure 12 is similar to Fig. 11 but using the calculated results for gear positions A_3 and A_7.

Obviously, Fig. 12 agrees very well with Fig. 11 although the experimental results are more scattered in general than the calculated results.

Figure 13 indicates for the series A how the radial conversion ratio $K_r=a_r/a_t$ and the axial one $K_z=a_z/a_t$ vary with the gear position zl, where the experimental values are obtained on the average from the figures for each gear position similar to Fig. 11, while the calculated values are those for the driving condition of $N=700$rpm and $T_Q=4\,000$kgmm. The relative value of a_t in Fig. 13 means the RMS value of the circumferential acceleration a_t at each gear position relative to a_t at A_1 or in case of gear position being just in the middle of bearings.

It is because of the complete release in the couplings between y and ϕ when the gear is set just in the middle of bearings, as has already described, that the calculated value for K_z in Fig. 13 is very small at $zl=110$mm. Excepting K_z near this region, where the effect of the static unbalance or the dynamic one and the non-linearity of the stiffness of bearings or the small difference in them and so on should be considered in addition to the above-mentioned analysis, the theoretical curves for the conversion

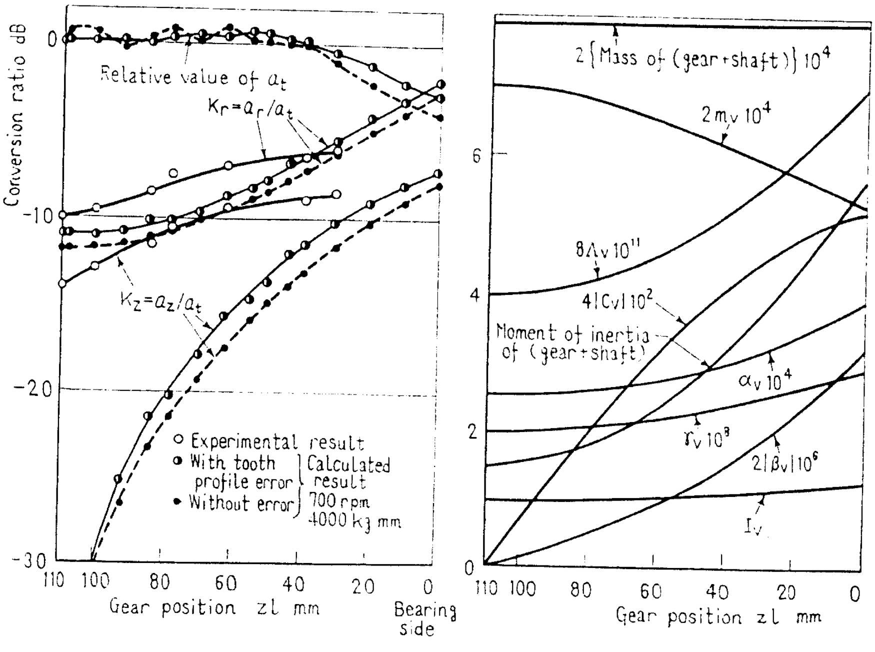


Fig. 13 Change of conversion ratios an calculated coefficients with gear position(Series A)

ratios may be in comparatively good agreement with the experimental ones. It is also obvious from Fig. 13 that the tooth profile error does not change the tendency of the conversion ratios with the gear position but it may produce some difference in the levels, which could be ascribed to the difference in the degree of influence of the tooth profile error on each directional vibration (see Fig. 4).

The abscissa of Fig. 13 indicating the distance between the test-gear and the nearer bearing, the conversion ratios are found to be nearly symmetric to the middle of bearings because the experimental values for A_4 and A_6 are almost on the same curves for A_1, A_2, A_3, A_5 and A_7, which can be clarified by the theoretical analysis when each bearing has the same properties.

Figure 14, the results for the series B similar to Fig. 13, indicates also that the calculated results for K_r coincide well with the experimental ones, especially in the tendency of transition with the gear position. On the other hand, each of the calculated curves for K_z shows a peak at some gear position dependent on the rotational speed, which is caused by the harmonic resonance between the natural frequency of ϕ_V and the contact frequency (the 5th, 4th and 2nd harmonic resonance for $N=700$, 1000 and 1350rpm, respectively) as is similar to the peak in a_z at $N=650$rpm in Fig. 10. This fact suggests there remains a problem to be solved relating to the value of the fraction of critical damping ζ or the method to represent the damping forces. Omitting these peaks and averaging 3 curves for K_z, however, the slope of the curve for the series B is more gentle than that for the series A just as for the curve K_r, which agrees with the experimental results.

The effect of the mass of bearings neglected in this paper may be larger in the series B than in the series A, the ground of which is that the main frequency of the gear vibrations is higher in the series B because the radius of gear-shaft is smaller there and so is the polar moment of inertia. This may explain why the difference between the experimental values of the conversion ratios and the calculated ones is comparatively larger in the series B than in the series A.

There is indicated in addition in Figs. 13 and 14 how the values, such as the elastic constants α_V, β_V, γ_V and Λ_V and the equivalent mass m_V, the equivalent moment of inertia I_V and the dynamic coupling coefficient C_V, change according to the gear position. Comparing the left figure with the right one in Figs. 13 and 14, respectively, K_r is seemingly related largely to m_V, and K_z to β_V, C_V, but the further details will be reported at a later date.

5. Summary

As described above, we analyzed theoretically the generation mechanism of the radial vibration and the axial one of spur gears and compared the calculated results with the experimental ones.

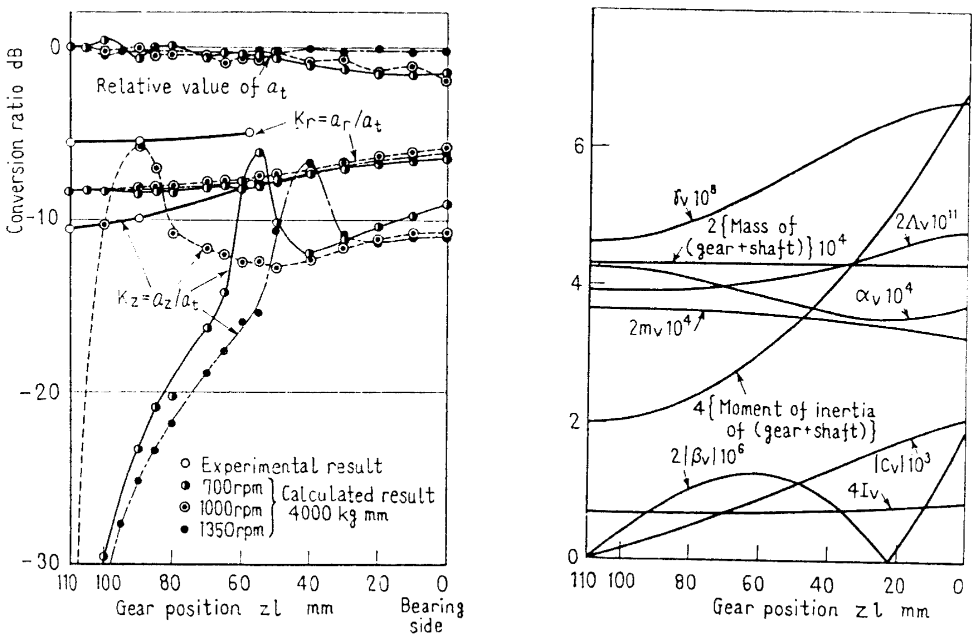


Fig. 14 Change of conversion ratios and calculated coefficients with gear position (Series B)

The results are summarized below.

1) Three-directional vibration of spur gears is represented by a mutually coupled and parameter-excited vibration system, i. e., the circumferential vibration and the radial one are coupled statically and the axial vibration is coupled both statically and dynamically with the radial one, and the coupling coefficients are determined by the gear teeth stiffness and the several constants peculiar to the shaft-bearings system. And the radial vibration and the axial one are generally stable if the circumferential vibration is stable.

2) It is not necessary in general to consider the gyroscopic action.

3) The values of three-directional vibrations change generally with the gear position relative to bearings, the tendency of which differs according to the combination of the stiffness of mating gear teeth, shaft and bearings and the mass of the shaft.

4) Therefore, the conversion ratios of the radial vibration and the axial one from the circumferential vibration depend on the combination of gears, shafts, and bearings, too. The details are the subjects for a future study.

5) As for the radial conversion ratio, the vibration model derived in this paper can give a qualitatively good explanation for the experimental results. But the effects which are not considered here, such as unbalances and the non-linearity in the properties of bearings, seem to be large factors for the axial conversion ratio.

References

(1) Aida, T., et al., *Trans. Japan Soc. Mech. Engrs.* (in Japanese), Vol. 34, No. 268 (1968-12), p. 2254.

(2) e. g., Imachi, I., et al., *Mechanical Vibration* (in Japanese), (1970), p. 194, Asakura Ltd.

(3) p. 2226, in ref. (1).

(4) Aida, T., et al., *Trans. Japan Soc. Mech. Engrs.* (in Japanese), Vol. 35, No. 278 (1969-10), p. 2113.

(5) p. 2237, in ref. (1).

Discussion

Y. Terauchi (Hiroshima University):

(1) The axial vibration being explained by the rotational displacement ϕ of the gear-shaft here, there can exist another type of axial vibration, such as the oscillation of gear body as a whole or the oscillation like a saucer in its form, where every point on the gear body oscillates in phase as has been recognized in the experiments on the phase relation of the axial vibration by questioners. How do you consider this point?

(2) The authors describe "It is due to a larger deflection of the bearing-metal … that K_p is small" on the 13 th line of right column at page 1100, but I wonder if the deflection of bearing-metal is so large.

M. Utagawa (Hitachi Ltd.):

(3) The theoretical result that the axial vibration cannot be induced when gears are set just in the middle of bearings as on page 1099 is contrary to both the empirical fact and the experimental one (out of coincidence near the region of $zl = 110$ mm in Fig. 13). Among the neglected factors in this theoretical analysis, what does the author suppose the main factor that can explain the above is?

For example, the elasticity of the gear body itself as a circular disc or the tightness between gear and gear-shaft could, the discusser thinks, be suggested as the factor.

(4) Comparing Fig. 10 with Fig. 9, the calculated results agree in general with the experimental ones as for their tendency but at some points there exists a difference in level up to about 10 dB, which means a few times the difference in absolute value. Therefore, the discusser apprehends that the more important factors than the effects of shaft and bearings treated here would be missed in this paper.

K. Fujita (Okayama University):

(5) What do you intend to point out about the vibration mode which is excited by the axial vibration? Does it mean the vibration of the gear body as a circular disc? I think the axial vibration due to elasticity of gear body would be induced by the vibrations of a gear system and the relation between noise and vibrations of a gear system would become much more complicated.

(6) Although all natural frequencies of three-directional free vibrations have almost the same value in this experiment as in page 1101, they do not coincide in general with each other. Therefore, the relation between accelerations of each directional vibration and the revolving speed as in Figs. 9 and 10 would be more perplexing with numbers of peaks corresponding to the degree-of-freedom. What is your opinion about this matter?

Authors' closure

(1) The axial vibration, which cannot be explained only by ϕ as pointed out, has been de-

tected in the other experiment by authors, too. These facts indicate the necessity to consider in addition that the gear body is also elastic. It is not impossible to solve the equation of vibration for the so-called distributed-parameter-system of the elastic body, although there is a definite difference between the method to solve it and the method to solve the equation for the lumped-parameter-system treated here. But the most troublesome problem is how the disturbing forces or the disturbing displacements, which may act in the axial direction of spur gears, are induced. It is clear from the experiments by questioners or authors that such disturbing forces or displacements sometimes play a weighty part in the generation of the axial vibration of spur gears. For instance, authors can list up the disturbing force resulting from the friction forces on mating gear teeth or the disturbing displacements supplied to gear-shafts from outside of the gear system such as the driving apparatus and the driven one, but there are involved many factors hard to evaluate quantitatively in the theoretical calculation, especially in the latter. So it might be very difficult to estimate such a type of axial vibration quantitatively by the calculations on the basis of the results of only the static test for the given spur gear apparatus.

(2) In order to get values for the stiffness of shaft and bearings, the deflections are measured at the bearing-pedestals, the flanges of the bearing-metals and several points with appropriate distance on the gear-shaft. According to the experiment, a large discontinuity was recognized between the deflection of the bearing-metal and that at the very nearest point on the gear-shaft to the bearing-metal, which was interpreted to be caused by the elastic deformation of the bearing-metal, but the shear deflection of the shaft may be also a cause of it. This difference in the deflections was about a few times as large as the deflection of the bearing-metal which was nearly equal to that of the bearing-pedestal; in other words, it resulted in about 70 % of the deflection which determined the bearing stiffness in Table 1. Moreover, the calculated values for deflections of the bearing-pedestal and the bearing-metal based on the approximate elastic theory are several ten times as small as the experimental values.

(3) The matter pointed out has been discussed many times in the midst of the theoretical analysis and authors determined to make it their basic attitude to explain the generation mechanism by use of a vibration model as reasonably simple as possible, because of the limit in time available to calculate. So the authors would like you to understand that the factors taken into consideration here are also based on fairly rough assumptions.

As for the reason why such result has been derived from the theoretical analysis, the following factors which are ignored here may be pointed out besides the suggested matters [(a) the vibration of gear body as a circular disc, (b) the tightness between gear and gear-shaft]: (c) non-uniformity in lead error, (d) non-linearity or the effect of dynamic properties of the stiffness of bearings and non-uniformity of them according to each bearing, (e) the effect of vibrations of masses besides those of gears and shafts, (f) the effect of dynamic unbalance and (g) the disturbing forces or displacements which act possibly on the gear-shafts from the outside apparatus in relation to (a) and (e), too.

Concerning (a) and (g), the above (1) would be referred to. The effect of (b) may be considerably large when the gear and gear-shaft do not consist of one body as in this experiment, but it is hard to take (b) into the theoretical equation. This effect may additionally result in such a fact that the equivalent radius of shaft is smaller than the actual one in the series A. The effect of (c) cannot individually satisfy the experimentally obtained condition that each of the main frequencies of three-directional vibrations is same, so far as the gear is set just in the middle of bearings. Accordingly, the effect (c) may play some part when it is combined with other factors. Even if the gear is set just in the middle of bearings, the coupling coefficients between y and ϕ will not generally be equal to zero when the effects of (d) and (e) are considered, and the axial vibration will be affected by the circumferential vibration and the rotational frequency, owing to the dynamic unbalance (f). So these effects of (d), (e) and (f) are interpreted to be comparatively large in the axial vibration with one nodal diameter. Considering the fact that the other vibratory modes have been recognized, however, the effect of (a) would play an important part depending on circumstances.

(4) There is no alternative explanation of the large difference in level of the axial vibration, especially when the gear is set near the middle of bearings, because of the above-mentioned factors remaining to be considered. But the level difference in the radial vibration may be ascribed mainly to that in the circumferential vibration, referring to the fact that the calculated values for the radial

conversion ratio agree comparatively well with the experimental ones. And this may result from the non-uniformity of tooth profile errors in each pair of mating teeth and the effect of pitch errors. It may be suggested that the tooth profile errors used in the calculations cannot represent the reasonable average of the actual ones in the size and the form. Moreover, the gear teeth stiffness in this report is calculated based on the paper(*1) by Dr. Ishikawa. But according to his latest report(*2), the deflection of mating gear teeth due to contact pressure is estimated larger to reduce the stiffness by about 30 % than before. Therefore, if the stiffness is calculated by the paper (*2), the resonance number of rotations is also reduced and it is expected that the level of the circumferential vibration under same number of rotations would be estimated to be larger. The authors would like to indicate in addition that the following result has been obtained, i.e., the experimental value for the main frequency is nearer to the calculated value using the stiffness based on the paper (*2) than that on the paper (*1) in case of considering the coupled three-directional vibrations, although the calculated value based on the paper (*1) agrees well with the experimental value when we set an eye only on the circumferential vibration as before.

A part of the level difference of the radial vibration may be explained also by the dynamic properties of bearings which are neglected here.

(5) The axial vibration of gear in this paper, which is discussed by the rotational displacement ϕ of the shaft, is of the vibratory mode with one nodal diameter ($s=1$) both vertically and horizontally. Considering the fact that the gear body is elastic, as pointed out, it is necessary to solve the differential equation, $\rho b(\partial^2 w/\partial t^2)+D\Delta^2 w=0$ (where D is so-called flexural rigidity and the friction forces are neglected), which governs the motion of the elastic body, under such boundary conditions as $[w]_{r=r_0}=r_0\phi$, $[\partial w/\partial r]_{r=r_0}=\phi$, ($r_0$: radius of shaft), and so on, where ϕ is calculated in this paper. But the vibratory mode is limited to $s=1$ even if such procedure should be taken. Although the gear body is assumed to be rigid here, one of the reasons is that the method to solve the above so-called equation of motion with forced displacement of an elastic body differs fundamentally from the method to solve the differential equation mentioned in this paper. According to our experiment regarding the axial vibration of gears, the vibratory mode of $s=2$ has been recognized under some circumstances though the mode of $s=1$ is generally predominant. So it is clear that the so-called axial vibration as a circular disc is sometimes induced by many factors neglected here [refer to the above (3)] and the details are the subjects for a future study.

(6) The authors apologize for an expression which may lead to misunderstanding. The questioner might understand that natural frequencies of three-directional vibrations have close values to each other from the sentence "this main frequency is … the nearest to it among those of three-directional free vibrations" and the wave forms shown in Figs. 7 and 8, but each of natural frequencies of each directional free vibration has a different value as pointed out. However, finally the gear oscillates at one main frequency regardless of its vibrational directions as in Figs. 7 and 8, and that sentence only means that the main frequency is the nearest to the natural frequency of the circumferential free vibration among those of x, y_I, y_{II}, ϕ_I and ϕ_{II}. Therefore, it is quite all right to presume that the resonances between natural frequencies of each directional free vibration and the contact frequency would occur at a different number of rotations, respectively, and the more complicated aspect would be presented although the degrees of peaks might differ from each other according to the degree of couplings, if the experiments or the calculations were made for the region of higher speeds than in Fig.9 or 10.

(*1) Ishikawa, J., *Trans. Japan Soc. Mech. Engrs.* (in Japanese), Vol. 17, No. 59 (1951-7), p. 103.

(*2) Ishikawa, J., *Bulletin Tokyo Institute of Tech.*, No. 2, p. 199.